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A CONTRIBUTION TO THE THEORY OF FINITE DIFFERENCES.

By Jos. Kaucký.

This paper contains a short demonstration of equation (8) which was treated by H. L. Smith in his paper "On the Ampère-Cauchy derived functions".*

Consider a function $f(x)$ of a single variable which we suppose has a derivative of order n. Let $x_0, x_1, \cdots x_n$ be numbers differing from each other and let us form the following sequence of functions:

$$(1)\qquad \begin{aligned} &[x_0] = f(x_0), \qquad [x_1] = f(x_1), \cdots \\ &[x_0\,x_1] = \frac{[x_0]-[x_1]}{x_0-x_1}, \qquad [x_1\,x_2] = \frac{[x_1]-[x_2]}{x_1-x_2}, \cdots \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ &[x_0\,x_1\cdots x_n] = \frac{[x_0\,x_1\cdots x_{n-1}]-[x_1\,x_2\cdots x_n]}{x_0-x_n}. \end{aligned}$$

The expression $[x_0\,x_1\cdots x_n]$ may be written in the form of an integral. First it is evident that

$$[x_0\,x_1] = \int_0^1 dt_1\, f'[(1-t_1)\,x_0 + t_1\,x_1];$$

then by complete induction we get easily the general relation

$$(2)\qquad \begin{aligned} [x_0\,x_1\cdots x_n] = &\int_0^1 dt_1 \int_0^{t_1} dt_2 \cdots \\ &\int_0^{t_{n-1}} dt_n\, f^{(n)}[(1-t_1)\,x_0 + \cdots + (t_{n-1}-t_n)\,x_{n-1} + x_n\,t_n]. \end{aligned}$$

Suppose we are given n numbers $w_1, w_2, \cdots w_n$ and form the differences

$$\underset{w_1}{\Delta} f(x) = \frac{f(x+w_1)-f(x)}{w_1} \qquad (w_1 \neq 0)$$

and in general

$$(3)\qquad \overset{n}{\underset{w_1 w_1 \cdots w_n}{\Delta}} f(x) = \underset{w_n}{\Delta}\left(\overset{n-1}{\underset{w_1 w_1 \cdots w_{n-1}}{\Delta}} f(x)\right).$$

From this definition there follows first

$$\underset{w_1}{\Delta} f(x) = \int_0^1 dt_1\, f'(x + w_1\,t_1)$$

* These Annals, vol. 25 (1924), p. 124.

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